Efficient Algorithms for Robust Feature Matching

Mount, Netanyahu and Le Moigne

November 7, 2000

Presented by Doe-Wan Kim

Overview on Image Registration

- Where is it used?
  - Integrating information from different sensors
  - Finding changes in images (different time/condition)
  - Inferring 3D information from images where camera/object have moved
  - Model-based object recognition
- Major research areas
  - Computer vision and pattern recognition
  - Medical image analysis
  - Remotely sensed data processing

Registration Problems

- Multimodal registration
  - Registration of images from different sensors
- Template registration
  - Find a match for a reference pattern in an image
- Viewpoint registration
  - Registration of images from different viewpoints
- Temporal registration
  - Registration of images taken at different times or conditions

Characteristics of Methods

- Feature space
  - Domain in which information is extracted
- Search space
  - Class of transformation between sensed and reference image
- Search strategy
- Similarity measure

Introduction

- Approaches to image registration
  - Direct use of original data
  - Feature (control points, corners, line segment etc.) matching
- Algorithms for feature point matching
  - Branch and bound
  - Bounded alignment

Classification of Algorithm

- Feature space
  - Feature points from wavelet decomposition of image
- Search space
  - 2 dimensional affine transformation
- Search strategy
  - Branch and bound algorithm
  - Bounded alignment algorithm
- Similarity metric
  - Partial Hausdorff distance

Problem Definition

- A,B: point sets (given)
- T: Affine transformation
- Find the transformation τ that minimizes the distance between τ(A) and B
- Two errors
  - Perturbation error (predictable)
  - Outliers

Similarity Measure

- Distance measure between point sets needs to be robust to the perturbation error and outliers.
- Use partial Hausdorff distance

Partial Hausdorff Distance

For given two point sets A, B and $k, 1 \le k \le |A|$

$H_k(A,B) = K^{th}_{a \in A} \min_{b \in B} dist(a,b)$

For $0 < q \le 1$,

$H_q(A,B) = H_k(A,B), k = \lceil q|A| \rceil$

Definitions

Find τ that minimizes

$H_q(\tau(A), B) = sim_q(\tau)$

where $sim_q(\tau)$ is the similarity measure of τ

τ_{opt} : optimum transformation

sim_{opt} : optimum similarity

Definitions (cont'd)

ε_r : relative error bound

ε_a : absolute error bound

ε_q : quantile error bound

weak quantile : $q' = (1 - \varepsilon_q)q$

τ is approximately optimal if

$sim_{q'}(\tau) \le (1 + \varepsilon_r) sim_{opt}$ or $sim_{q'}(\tau) \le sim_{opt} + \varepsilon_a$

Definitions (cont'd)

- Cell
  - Set of transformations (hyperrectangle)
  - Represented by pair of transformations (τ_{lo}, τ_{hi})
  - Upper and lower bound of similarity
  - Active or killed
- Upper bound
  - Sample any transformation τ from cell and compute $sim_q(\tau)$

Lower Bound

- Uncertainty region
  - Bounding box rectangle for the image of a under a cell T
  - Defined by corner points $\tau_{lo}(a)$ and $\tau_{hi}(a)$
  - For a cell, each point of A has an uncertainty region
- Compute distance from uncertainty region to its nearest neighbor in B
- Take qth smallest distance to be $sim_{lo}(T)$

Uncertainty Region

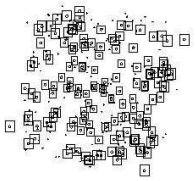

Figure 1: Uncertainty regions.

Cell Processing

- Split
  - Split cell so as to reduce the size of uncertainty region as much as possible
- Size of uncertainty region
  - Size of longest side
- Size of cell
  - Largest size among the uncertainty region
- Store cells in a priority queue ordered by cell size (the cell with largest size appears on top of priority queue)

Cell Processing (cont'd)

- Finding largest cell
  - Cell generating the largest uncertainty region
  - For transformation $\Delta = \tau_{hi} - \tau_{lo}$, and point a, find the largest absolute value among $\Delta_{ij} a_j$ and t_i Δ_{ij} for $i, j \in \{1,2\}$ and translation vector (t_1, t_2)

Branch-and-Bound Algorithm

1. Initialize priority queue with initial cell.
   Set $sim_{best} = \infty$
   Repeat 2 - 5 until queue is empty or $sim_{best} \le \varepsilon_a$
2. Remove the largest cell T from queue
   a) Compute uncertainty regions for every $a \in A$
   b) For each uncertainty region, compute nearest neighbor.
   c) Compute lower bound.
   If $sim_{lo}(T) > \frac{sim_{best}}{1+\varepsilon_r}$ or $sim_{lo}(T) > sim_{best} - \varepsilon_a$ return to 2.
   d) Compute upper bound.

Branch and bound algorithm (cont'd)

3. If $sim_{hi}(T) < sim_{best}$, update sim_{best}, τ_{best}
4. Split T into T_1, T_2. Compute size of each cell.
5. Enqueue T_1, T_2

Bounded Alignment

- Drawback of B&B: high running time
- Alignment
  - Triples from A are matched against triples from B in order to determine a transformation
  - can be applied when many cells have uncertainty regions that contain at most a single point of B
- Noisy environment
  - For a noise bound η, suppose that for each inlier a, distance between $\tau_{opt}(a)$ and its nearest neighbor is less than η

Alignment

Definition

For $\alpha \geq 1, a_1, a_2, a_3 \subseteq A$ is well-distributed relative to α, if every point $a \in A$ can be expressed as

$a = \alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3$ s.t. $|\alpha_i| \leq \alpha$

Lemma

For a well-distributed triple a_1, a_2, a_3 and τ maps a to a point within distance η of a_i, distance between $\tau(a)$ and a is at most $3\alpha\eta$

$$|\tau(a) - a| \leq \sum_{i=1}^{3} a_i |\tau(a_i) - a_i| \leq \eta \sum_{i=1}^{3} \alpha_i \leq 3\alpha\eta$$

Required Steps (after 2 (d) of B&B)

e) For each a, count number of points of B within uncertainty region.
If at most one, and NN is within η, this region is alignable.

f) If alignable fraction is less than q_s, return to 2.
Otherwise, Let $A' \subseteq A$ s.t. for each $a \in A, \exists b \in B$ lies either inside or within η of a's uncertainty region.
Repeat following N_s times.
i) Sample triples from A' until well-distributed one is found.
ii) Compute transformation and similarity.
iii) If similarity is better than sim_{best}, replace.
If simliarity for all N_s transformation exceeds sim_{best} by η, kill this cell.

Experiments on Satellite Imagery

- 3 Landsat/TM scenes:Pacific NW, DC, Haifa
- AVHRR scene: South Africa
- GOES scene: Baja California
- Parameter settings
  $\varepsilon_r = 0.1, \varepsilon_a = 0.2 - 0.4, \varepsilon_q = 0.2$
  $\eta = 0.4 - 0.6$
  $N_s = 10 - 20$
  q_s : s.t. $q_s |A|$ to be order of 100

Experiments (Pacific NW)

- Original image: 128 X 128 gray-scale image
- Transformed image: Artificially generated by applying -18° rotation
- |A|=1765, |B|=1845
- Target similarity: 0.81
- Initial search space
  - Rotation: 2°
  - X translation: 5 pixels
  - Y translation: 5 pixels

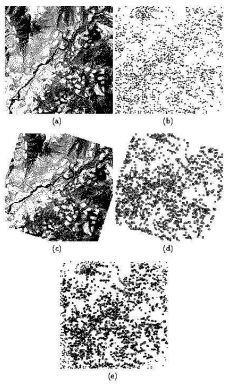

Figure 6: Matching two Landsat/TM images of the Pacific northwest.

Experiments (Washington, DC)

- Original image: 128 X 128 gray-scale image
- Transformed image: Generated by applying translation (32.5,32.5)
- |A|=763, |B|=766
- Target similarity: 0.71
- Initial search space
  - Rotation: 10°
  - X translation: 5 pixels
  - Y translation: 5 pixels

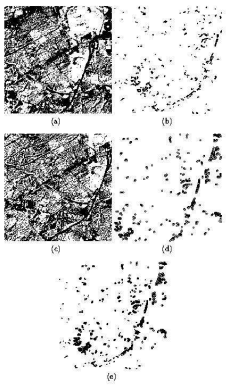

Figure 7: Matching two Landsat/TM images of Washington, DC.

Experiments (Haifa, Israel)

- Images taken on two different occasions
- |A|=1120, |B|=1020
- Target similarity: 0.5
- Initial search space
  - Rotation: 5°
  - X translation: 5 pixels
  - Y translation: 5 pixels

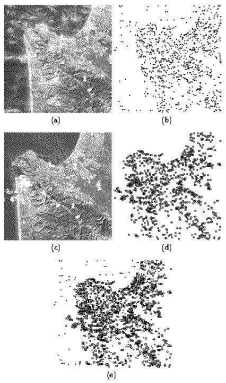

Figure 8: Matching two Landsat/TM images of Haifa, Israel.

Experiments (South Africa)

- Images are taken at two different times
- |A|=872, |B|=927
- Target similarity: 1.0
- Initial search space
  - Rotation: 10°
  - X translation: 5 pixels
  - Y translation: 5 pixels

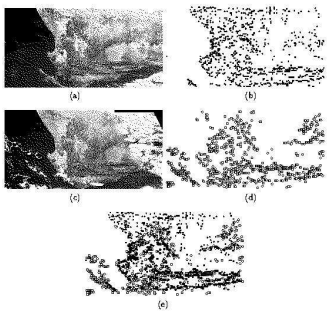

Figure 9: Matching two AVHRR images of South Africa.

Experiments (Baja, California)

- Images are taken at two different times
- |A|=326, |B|=503
- Target similarity: 0.0
- Initial search space
  - Rotation: 10°
  - X translation: 5 pixels
  - Y translation: 5 pixels

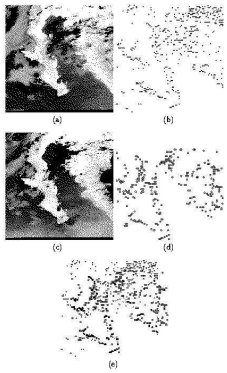

Figure 10: Matching two GOES images of Baja California.

Experiment Results

	Alg	Cells	Sec	Angle	t_x	t_y	Sim
Pac NW	B&B	399	32	18.9	0.34	−0.49	0.42
Pac NW	B. A.	7	3	18.8	0.10	−0.54	0.47
DC	B&B	385	13	−0.08	32.1	32.0	0.30
DC	B. A.	35	4.8	0.14	32.5	31.7	0.60
Haifa	B&B	1103	55	−0.06	13.6	−1.02	0.59
Haifa	B. A.	25	7.4	−0.23	14.1	−1.13	0.73
SA	B&B	2037	81	−0.005	4.1	−5.8	0.18
SA	B. A.	29	3.9	0.0	5.0	−6.0	0.00
Baja	B&B	1164	17	−0.16	9.9	−10.0	0.17
Baja	B. A.	11	0.6	0.0	10.0	−10.0	0.00

Conclusion

- Feature matching for image registration
- Use Partial Hausfdorff distance
- Branch and bound algorithm
- Bounded alignment algorithm
- Experiments on satellite images